Behavioral Despair in the Talmud

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Abstract: We solve two "unsolvable" (*teyku*) problems from the Talmud that had remained unsolved for about one and a half thousand years. The Talmudic problems concern the implied decision-making of farmers who have left some scattered fruit behind, and the alleged impossibility of knowing whether they would return for given amounts of fruit over given amounts of land area if we aware of their behavior at exactly one point. We solve the problems by formalizing the Talmudic discussion and expressing five natural economic and mathematical assumptions that are also eminently reasonable in the original domain. If we also allow a sixth assumption regarding the farmer's minimum wage, we can solve two other related unsolvable problems.

Introduction

Suppose we want to predict a major decision of a social unit (a person, an organization, a corporation) when a certain critical situation raises. Suppose we can describe the set of all possible decisions that can be made and we can describe all of the parameters that identify the critical situation.

We have a few reference points for which we know what decisions have been made in the past under an associated set of parameters. What would be the right approach to solve such a problem?

One possible solution would be to use some kind of statistical or computer analysis, such as discriminant analysis, cluster analysis, or some machine learning technique. That would require a large number of observations, approximately proportional to number of possible sets of parameters.

The Talmud suggests an approach that can be applied when the number of observations is small. And it starts with the simplest possible example: a decision that consists of only two values, and a situation that can be described by only two parameters. The Talmud, however, does not provide a complete solution; we offer the complete solution here.

The Talmud

The Babylonian Talmud is a compendium of Jewish law and associated discussion spanning between the first and fifth century of the common era. It comprises about sixty tractates covering six broad subjects. Typically, an oral law is memorialized and cited (the *Mishna*), and then relevant discussion about its meaning and scope is recorded (the *Gemara*). Tradition holds that the oral law was given alongside the written law (*Torah*) and was passed down by generations until it was finally written down and eventually became the core of the Talmud. In addition to recording the accepted ruling, the Talmud is unique in recording all sides of the debate, including minority and rejected opinions. For more information and history of the Talmud, c.f. Steinsaltz (2006).

Some situations do not have a ruling and are explicitly listed as *teyku*. Such passages occur 319 times in the Talmud and are listed by Jacobs (1981). The meaning of *teyku* is not definitively known. Its literal meaning is akin to “it stands,” in other words, the problem is unsolvable. On the other hand, another tradition suggests it is an acronym for a phrase meaning that the Biblical prophet Eliyahu will come and solve these problems at the time of the coming of the Messiah. Another interpretation is that *teyku* means the pros and cons of the situation are equally balanced by definition. In our context, it is clear the meaning of *teyku* is most naturally read as “unsolvable” due to a seeming inability to predict a person’s behavior. Since we now provide a logically connected way to actually do so, these particular problems are no longer *teyku* – they are solved.

To our knowledge, these are the first unsolved problems in the Talmud to be solved in the 15-20 centuries since its publication. The Talmud has gone through innumerable editions and been the subject of countless commentaries. The standard commentaries, now included in traditional printings of the Talmud, include the famous French commentator Rashi and his in-laws, grandchildren, and great-grandchildren, among others, stretching from about 1000-1200 CE. Additional commentaries also typically line the Talmud, and many other commentators and codifiers exist independently of the Talmud and are often referred to by abbreviated nicknames, c.f. Rosh (Asher ben Jehiel), RaN (Nissim ben Reuven), Rabenu Chananel (Chananel ben Chushiel), and Rambam (Moses ben Maimon, a direct ancestor of two of the authors of this paper).

bBava Metzia 21a

bBava Metzia concerns what we modernly call tort law or damages, specifically property law, the responsibilities of borrowers and lenders, and the question of lost property. Chapter 2 focuses on lost property and when it may or must be returned to its owner, and when it may be kept.

The cited *Mishna* notes that, among other things, “scattered fruit” may be kept by the finder. The *Gemara* then discusses how much fruit over what area is considered scattered. (As an aside, one conclusion is that the fruit must appear random; fruit piled up in an ordered fashion can not be considered scattered.)

The principle behind the question of whether the fruit belongs to the finder or the original owner rests on the concept of “despair” or lost hope. Any article is considered property of the finder if the original owner has despaired of ever finding it. In the case of scattered fruit, the concept of despair essentially means the owner disowns the fruit: even though he may know precisely where it is, he will choose to never return to collect it. So the Talmud asks, how much fruit over how much land can we assume, as a matter of law, represents this kind of despair on the part of the owner? More specifically, when can the finder of fruit properly infer that the owner has despaired of the fruit and left it behind, and when is the owner likely to return and collect it?

The first specific pronouncement comes from R. Yitzhak, where the text of the gemara reads in relevant part:

> *The Mishna records: "If he found scattered fruits" [the finder may keep them].*
>
> *How much (fruit scattered over how much area)?*
>
> *R. Yitzhak said: One kav over an area of four amot.*

What is a *kav* and what are *amot*? We do not know for sure but standard thought is that a *kav* is a measure of volume of about two quarts, and an *amah* (singular) or *amot* (plural) refer to length, or, in our context, area. An *amah* is about a square foot and a half to two square feet.

Next, section 21a of bBava Metzia lists several reasonable questions related to this quantitative definition of scattered fruit, questions that the Talmud ultimately declares *teyku*, unsolved and perhaps unsolvable:

> *R. Yirmiah asked: What of half a kav over a two amah area. Is it the case that a kav spread over four amot belongs to the finder because the collection effort is too great (so that the owner abandons the stuff) -- so that, in the case of half a kav over a two amah area, which does not involve so much work, the owner does not abandon his rights and the finder cannot claim the grain? Or perhaps the rationale for the kav per four amot ruling is because a kav simply isn't enough to be concerned with -- in which case, a half kav is certainly insignificant, and the owner _does_ renounce ownership (and the finder can claim it)?*
>
> *R. Yirmiah's second question: Two kavs over an eight amah area -- what is the law? Is the kav per four amot rule because the collection effort is too great, in which case the collection effort in an eight amah area is certainly excessive, and the owner abandons it (and it belongs to the finder). Or perhaps the kav per four square amot rule is because a kav is simply an insignificant amount, but two kavs _are_ significant (and the owner does not renounce ownership)?*

Results

In this paper we answer these first two questions of R. Yirmiah[1]. (N.B.: R. Yirmiah asked additional questions, which we do not attempt to answer here, though we address some more of his questions in the appendix.) We answer them by stating explicitly some of the intuitively natural assumptions of the problem and analyzing the problem in a more formal setting. We visualize the problem with a graphical depiction of the decision choices faced by the owner, combined with natural conditions based on *kal va-chomer* (an a fortiori argument) and implied mathematical constraints. We show that both questions have a definite answer for all owners, and that the answer is: the fruit belongs to the finder in the first case but not in the second case.

Literature Review

In addition to the Talmud itself, its written commentaries over the centuries, and continuing debate among all who read it even today, there have been several academic papers dealing with issues related to the Talmud. To our knowledge, none have solved a *teyku* before. However, several have addressed various seeming puzzles. Most famously, Aumann and Maschler (1985) showed that the answer to a bankruptcy problem described in the Talmud corresponds precisely to the nucleoli

[1] One of R. Yirmiah's legacies in the Talmud is his expulsion from the house of study, presumably because of his propensity to ask either too many questions or questions with no answer or questions focused on seeming minutiae, c.f. Steinsaltz (1964). In the present case, however, R. Yirmiah's questions were *exactly* the right ones to ask, and in answering them, we hope to provide at least some evidence that his questions in general may have been more valuable than they are typically given credit for.

of coalitional games. Maymin and Maymin (2013) showed how an estate allocation algorithm of Maimonides, in his commentary on the Talmudic bankruptcy law, can be applied to the modern risk parity portfolio management strategy. Kleiman (2003) described the economic insights of three problems analyzed in the Talmud—public goods, interest rates, and compensation for pain—to show that the ancient scholars were both economy and market oriented. Katz and Rosenberg (2008) and Rosenberg (2008) show that the Talmudic laws relating to theft and fire damages, respectively, conform to economic principles of optimal incentives.

What does it mean for a *teyku* to be solved? As this is the first known solved *teyku* of all time, it is a relevant question to consider. There are two different kinds of answers. One is the standard academic answer that an open problem has been solved, and this is notable and interesting for the sake of knowledge itself; the fact that these problems have been open for several orders of magnitude longer than most solved open problems makes it all the more remarkable.

The other answer is one of practical import. For those who live by Talmudic laws and principles, how should their behavior change, if at all? On the one hand, there is a body of law of what to do when the law is unknown. Maimonides (c. 1180) for example states that in the *teyku* cases of R. Yirmiah that we are discussing here, one should not take the fruit, though if one does happen to take it, one is not thereafter obligated to report it. Meanwhile, another more speculative argument, extending a ruling of Rabeynu Chananel in bBava Metzia 24a, suggests that one *can* collect. On the other hand, now that we know for sure that we can reasonably conclude that the owner will not return, should the hypothetical fruit collector who had followed the view of Maimonides now change his behavior? The answer to this question touches on the age-old philosophical question of modeling in economics, most famously answered by Friedman (1953) in his example of the model of the billiards player that is too complicated for the billiards player to believably follow, yet which predicts his behavior accurately. A similar question needs to be addressed here: presumably the discussion in this paper was not obvious to anybody for thousands of years, so we cannot reasonably assume that people were following this precise logic. But is it the case that people tended to act in accordance with these predictions anyway? The answer to this question would help form the basis to a decision as to what the law now ought to be. What we are doing is putting out the solution to the *teyku*. The ultimate decision of whether or not the code of law should change is beyond the scope of this paper.

Formal Statement of the Problem and its Solution

We assume, as does R. Yirmiah in his questions, that R. Yitzhak's statement is true for all owners: if one *kav* of fruit is scattered over an area of four square *amot*, the owner is presumed to have despaired of recovering the fruit, and whoever finds it can keep it.

In general, we want to know if an owner despairs if there are k *kav* of fruit over a square *amot*. Thus we want to know the nature of the function $f_i(k, a)$, which can take the value zero indicating despair, or one indicating no despair, meaning the fruit still rightfully belongs to owner i.

By *kol va-chomer*, we can clearly see that an owner that despaired of one *kav* of fruit over four square *amot* must *a fortiori* have despaired for any lesser amount of fruit, or for any greater amount of area.

Note that every individual owner i might have a different function f_i, so that what might be worth recovering for one person would not be worth it for another. However, for any given individual, the function $f_i(k, a)$ always has a value of either zero or one for any given values of k and a. How do we know this? Because an owner either will return to collect his fruit, or he will not. There is no middle ground, and no possibility of multiple answers.

A1 Assumption of Decision: $f_i(k, a) \in \{0,1\}$ for all k, a.

We know from R. Yitzhak's statement that $f_i(1,4) = 0$ for all individuals i.

R. Yirmiah's two questions imply that we do not necessarily know either $f_i(1/2,2)$ or $f_i(2,8)$ for all i; in other words, different individuals may have different answers to these questions, and therefore these questions must be *teyku*, unsolvable. Let us now solve them.

Consider the two-dimensional plot with area *a* running along the x-axis and the quantity of fruit *k* running up the y-axis depicted in Figure 1. The dark red point at $k = 1, a = 4$ represents the owner despairing of his fruit per R. Yitzhak's statement.

Note that the owner will *also* despair if there is less fruit and more area to cover. In general, if we know that $f_i(k^*, a^*) = 0$ for some specific amount of fruit k^* and land area a^*, meaning that the owner will despair at those levels, then we know that the owner will also despair for any other values that correspond to less fruit and more area. Specifically, we know that the owner will despair if there is less than one *kav* of fruit in four square *amot* or if one *kav* of fruit is dispersed on more than four square *amot*. Why? Because if he despairs of one in four, how much more so will he despair of one-half in four, or one in five.

This additional region of despair is depicted in Figure 1a as the lighter red region to the bottom and right of the dark red point corresponding to R. Yitzhak's statement.

Similarly, we know that if you do not despair, if you continue to assert ownership of *k kav* of fruit on *a* square *amot*, how much more so if there is more fruit, or if it is on a smaller area.

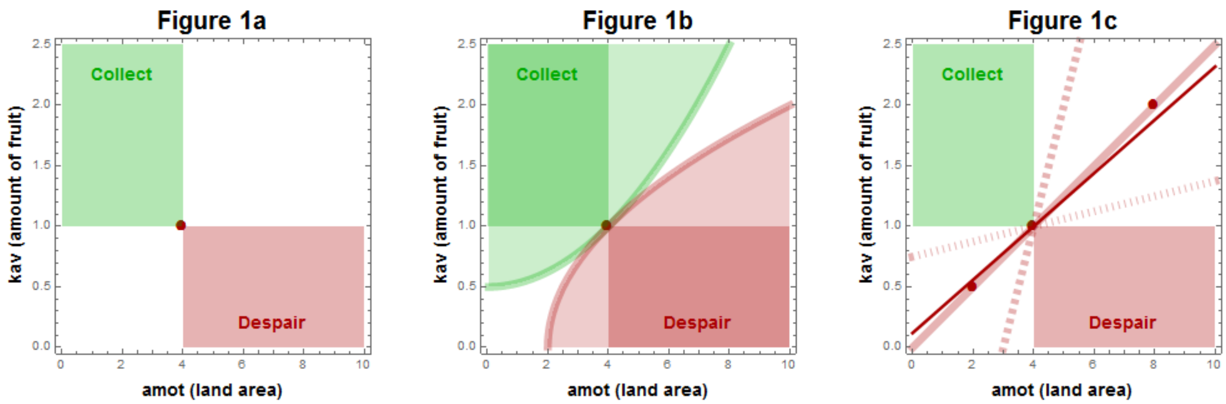
A2 Assumption of More-is-Better, Less-is-Worse:

$$f_i(k^*, a^*) = 0 \text{ implies } f_i(k, a) = 0 \text{ for all } k \leq k^* \text{ and } a \geq a^*$$

$$f_i(k^*, a^*) = 1 \text{ implies } f_i(k, a) = 1 \text{ for all } k \geq k^* \text{ and } a \leq a^*.$$

R. Yitzhak's statement implies a separation, namely that for more than one *kav* of fruit on less than four square *amot*, an owner will *not* despair. This continuing ownership claim is depicted as the light green region to the left and above the critical dark red point corresponding to R. Yitzhak's statement.

Thus, Figure 1a graphically represents R. Yitzhak's statement and its logical implications.



What about the two empty areas in the bottom left and top right? This is where we imagine individuals can differ. Those for whom the collection effort is too great will despair of the top right section but not of the bottom left; those for whom the quantity is insignificant will despair of the bottom left section but not the top right; hence, it seems, *teyku*. We seemingly cannot have a single law for all people.

However, we actually *can* make some statements about all people. What distinguishes one person's decision-making from another, in this context? Ultimately, it is just a separation between areas where they would despair and areas where they would not. Figure 1b shows a couple of examples. Perhaps an individual will not despair along the green curve, but will despair along the red curve.

As we know from above, these curves must be non-decreasing. Further, the areas the curves draw out must be closed: graphically, if you do not despair along the green curve, then you must not despair anywhere to the left or top of the curve, and if you despair along the red curve, then you must despair anywhere to the right or bottom of the curve.

With one reasonable assumption, those curves must each represent convex sets.

With k as the number of *kav* of fruit and a as the number of *amot* over which the fruit have been scattered, based on the formulation of the problem, we can reasonably assume that:

A3 Assumption of Scale: The set of all possible pairs of k and a under consideration is:

$$R_2^+ = \{(k,a): a \geq 0, k \geq 0\}$$

This assumption can also be naturally relaxed to finite bounds. For each individual owner, let's define $D \subset R_2^+$ as the despair set, a set of all pairs (k, a) where the owner will not collect the fruit, and $C = D'$ its complement, the set of all pairs (k, a) where the owner will collect the fruit.

Our main assumption is midpoint continuity:

A4a Midpoint Continuity: If $(k1, a1)$ and $(k2, a2)$ belong to D (or C), then the midpoint between these two points $((k1 + k2)/2, (y1 + y2)/2)$ also belongs to D (or C, respectively).

We can equivalently assume:

A4b ϵ-Midpoint Continuity: Given a small $\epsilon > 0$, the midpoint continuity assumption holds for all segments with length of the segments between $(k1, a1)$ and $(k2, a2)$ less than ϵ.

Midpoint continuity seems reasonable in the original setting, the idea being that if you would despair (or collect) at two points, you would also despair (or, respectively, collect) at their average point. However, one might argue that for very large distances between points, some aspect of psychology or irrationality might be a legitimate concern. To address this possible concern, ϵ-midpoint continuity is an even weaker assumption and even easier to justify, since it restricts the distance between two points to be arbitrarily small, and there cannot be any claim of any psychological hurdles at large numbers or similar counterarguments.

It's easy to show by separating an arbitrary length segment into those less than or equal to ϵ that midpoint continuity and ϵ-midpoint continuity are equivalent. In other words, either assumption A4a or A4b is sufficient for our purposes. (Note also that assumption A2 follows from A3 and A4, but we have left it in for ease of exposition.)

Any set satisfying midpoint continuity is called "midpoint convex." According to the Sierpinski (1934) theorem, midpoint convexity is equivalent to convexity, so sets *C* and *D* are convex.

The boundary of *C* and *D* must therefore lie on a straight line. That follows from the fact that *C* and *D* are convex sets and therefore according to the Minkowski theorem (c.f. Boyd and Vandenberghe, 2009), there should be a line separating them. As one set is a complement of the other, that line is the boundary for each of them.

So what about the potential area between the two curves? In Figure 1b, there is a gap between the green curve and the red curve; what happens there? As discussed above, there is no possibility for an owner to neither despair nor not-despair. One and exactly one of those conditions must obtain for every possible amount of fruit over every possible amount of land. Therefore, we now know that the two curves must intersect at every point. There can be no gaps. The only intersection of a convex and a concave curve is a straight line.

Therefore, the only possible decision making differentiation among individuals is what kind of straight line explains their choices. Each straight line must go through R. Yitzhak's point with despair, and it must intersect either along the y axis below 1.0 or along the x axis below 4.0. Figure 1c shows three possible examples: one that scales up and down proportionately (marked by the solid red line originating from the axis), one that is nearly horizontal and approaches the decision-making of the significance-centric person, and one that is nearly vertical and approaches the decision-making of the effort-centric person.

Note the two additional dark red points in Figure 1c, corresponding to R. Yirmiah's two questions. It appears that, while all lines will pivot on R. Yitzhak's point, some will go above the higher point and some will go below; some will go above the lower point and some will go below. So it appears as if we have made no progress, except to visually depict the Talmudic discussion.

But in fact we have already made a breakthrough. Notice that there is only one way for a single line to despair on both of R. Yirmiah's points, namely, the straight line emanating from the origin. For every other decision-maker, one despairs at one point, but not the other. Certainly there is no way for a single individual to assert ownership in both of R. Yirmiah's points while despairing at R. Yitzhak's point.

Will a reasonable person collect zero or a de minimus (*shaveh perutah*, lit., the value of the minimum coin, deemed worthless by the Talmud, approximately one penny's worth) amount of fruit over some non-zero land area? We contend that they will not, by definition of what it means for something to be de minimus.

A5 Despair Over Negligible Fruit: All owners will despair over a de minimus amount of fruit regardless of the amount of land area: $f_i(\epsilon, a) = 0$ for all a.

Therefore, of all the possible lines that a decision-maker could use to represent their choices in this situation, none of them may be like the dashed nearly-vertical line in Figure 1c.

Therefore, all individuals choose a line that passes through R. Yitzhak's point and some point on the y-axis between zero and one, such as either the nearly-horizontal dotted thick light red line or the nearly-diagonal thin dark red line in Figure 1c.

This means that they will *always* despair at R. Yirmiah's lower point of half a *kav* on two square *amot,* and they will *never* despair at R. Yirmiah's higher point of two *kav* on eight square *amot*.

And of course, the same statements are true for all points on the segment connecting the origin with the lower point, not just $a = 2, k = 1/2$, and similarly for all points in the continuation of this line, not just $a = 8, k = 2$.

The only exception, again, are people whose lines pass through the origin.

Will a reasonable person spend a de minimus amount of effort to collect a de minimus amount of fruit over a de minimus amount of land area? Here we again contend that they will not, because the de minimus return falls short of motivating an owner, and is by definition ownerless. Indeed, this follows by assumption A5 for $a = \epsilon$.

Therefore, we can rule out the straight line originating at the origin.

We have thus shown that every owner may differ in their decision making, but their decision making can always be represented by a line, that the line will cross at least slightly above the origin and also intersect R. Yitzhak's central point, and therefore *every* decision maker will always despair for R. Yirmiah's lower point and not despair at the higher point. And this result doesn't depend on the precise value of the de minimus amount.

It can be easily shown that, in order to despair, the maximum amount of *kav* on eight square *amot* should be two minus the de minimus amount, and in order to collect, the minimum amount of kav on two square amot should be one half plus half of the de minimus amount.

Extensions

We have shown that every decision maker can be represented as a straight line connecting R. Yitzhak's point and some minimum threshold point for how much profit a person needs to motivate them to collect the fruit over a negligible area of land. This means we can answer the two *teyku* problems in the Talmud regarding R. Yirmiah's points, but not necessarily other points. If we know the minimum wage of the person, however, we can more specifically infer his exact line, and be able to correctly predict whether he will despair or not at *any* point.

A6 Minimum Wage: Every person has a minimum wage that is common knowledge:

$$\forall i \exists w: f_i(w, 0) = 0 \text{ and } f_i(u, 0) = 1 \text{ for } u > w$$

Suppose the fruit was scattered at a certain distance from the farmer, where we can express the distance in terms of roundtrip time. If it would take the farmer an hour to come back, collect the fruit over a zero land area, and return to where he is, then it is reasonable for the farmer's opportunity cost to represent his minimum threshold. Although modernly wages are set by market prices and can fluctuate, in Talmudic times, wages and prices were relatively stable and widely known, as were distances to other towns or markets. Therefore, one who happened upon some scattered fruit could also be expected to know both the typical wage of the farmer who owned the fruit and how far away he is likely to be. Therefore, he can know the farmer's *exact* decision line, and know with relative certainty whether the farmer is likely to despair or not over the fruit.

This calculation will depend on several factors, including the distance and the wage, but also the rotting time of the fruit. R. Yirmiah asked several other questions including what happens in the event the fruit are pomegranates instead of apples, or if the fruit are grains of various varieties, and so on. With this additional assumption of the ability to calculate the opportunity cost of the farmer, a more specific and complete answer can be reached, in all cases.

Specifically, continuing the quotation of the Talmud from above:

> *R. Yirmiah's third question: A kav of sesame seeds spread over a four amah area -- what is the law? Is the Mishna's kav per four amot because grain is inexpensive, but sesame seeds are more valuable, so that the owner would not abandon them? Or perhaps the Mishna's reason is that the effort to collect the grain is too great -- and sesame seeds are even more difficult to collect, so the owner certainly would abandon them?*
>
> *R. Yirmiah's fourth question: A kav of dates over a four amah area -- what is the law? Is the reason for the Mishna's kav of grain per four square amot ruling because the grain is inexpensive? -- if so, a kav of dates over four amot or a kav of pomegranites over four amot is also inexpensive, and the owner will abandon them. Or perhaps the reason for the ruling on grain is that the collection effort is great, but a kav of dates or of pomegranites over a four amah area is not difficult to collect, so the owner does not abandon them?*
>
> *The gemara answers: Teyku.*

With the assumption of minimum wage, both of those additional questions by R. Yirmiah can now be answered. The change from one type of fruit to another, or to seeds of various types, can affect one or two things: it can affect the value of the fruit to be gathered, so if we view *kav* as a measure of value and not simply volume, then no change needs to be made; and it can affect the minimum wage if certain kinds of fruit or grain farmers have higher or lower opportunity costs. In Talmudic times when such wages were fixed and known, or more generally in any situation where such wages are fixed and known, and even then only to the extent that the minimum wage can be reasonably estimated, we now have the result that any person who finds any fruit can definitively tell whether it is scattered enough for it to belong to them, should they wish to take it.

Thus, without assumption A6, we solve the first two *teyku* in this portion of the Talmud; with assumption A6, we solve all four.

Relation to Traditional Economics

With assumption A6, each farmer's decision becomes a very simple combination of the farmer's minimum wage and marginal product. Every farmer's despairs at or below, and collects above, some line:

$$k = ma + b$$

In our terminology above, the b is his fixed cost minimum wage and m is his minimum number of marginal *kav* per marginal *amot*.

Relation to Behavioral Economics

People do not always act consistently or logically. The *Tosafot*, a medieval-era commentary on the Talmud, argues[2] regarding bBava Metzia 21a that, essentially, people may fail to act optimally but may instead act according to simple heuristics. One such heuristic it mentions is refusing to work for more than a given amount of labor, similar to the findings of Camerer, Babcock, Loewenstein, and Thaler (1997) that New York City taxicab drivers only work up to a target and then quit. Another heuristic is the psychological hurdle of leaving a job half-finished, similar to the aversion expressed by consumers against incompletely shaped objects in Sevilla and Kahn (2014).

Conclusion

One major insight, provided by the Talmud, is that the observation point that provides the most information about the decision making of a given individual or entity lies on the boundary between the two values of the possible decisions.

Under a couple of additional assumptions—a decision is made at every point in situation space, and the sets in this space corresponding to one specific decision value are convex—it is clear that

[2] The Tosafot offers two comments on the issue at hand. First, the Tosafot claims that R. Yirmiah's question is about half of the area of R. Yitzhak's 4x4 *amot*; in other words, that R. Yirmiah is asking whether a person would return to collect half a *kav* over an area of 4x2 *amot*. In our terminology, this would equate to an area of $\sqrt{8} \approx 2.83$ in the graph, which, for half a *kav*, we have shown would always result in despair. The second comment of the Tosafot is that R. Yirmiah is asking about an area of 2x2 *amot*. This is the same point as we assumed for R. Yirmiah, and hence also always results in despair.

we can predict the decision of every individual if we know two points on the boundary of the decision sets.

The surprising beauty of the example discussed in the Talmud is that even if we know just one decision point on the boundary, we can make an exact prediction for the person's decision making in some important cases, and these important cases are exactly the ones that are both raised and discussed in the Talmud. The main contribution of this paper has been in solving these cases.

Thus, using a handful of natural formal assumptions that are also reasonable and implicit in the original domain, we have solved two open problems in the nearly two-millennia-old Talmud that had not been solved since and were indeed marked as "unsolvable."

Other related commentary in this Talmud section could also be addressed through this approach. Another question raised by R. Yirmiah is, what happens if the fruit are pomegranates instead of apples? Or what if the fruits are seed; does it matter what kind of seeds they are? For such questions of quality, we can simply redefine the amount of fruit to represent the market value of the fruit.

For other questions, such as the speed of rotting of the fruit, we could extend the analysis and Figure 1 to a third dimension involving the fixed amount of time the owner has to return.

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